How Much Can One Trust Foundry's 3-Sigma PEX Corners?

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Outline

- What is a 3-sigma PEX corner?
- Inaccuracy of 3-sigma PEX corners
- How to improve the accuracy of wire resistance's 3-sigma corners?
- Future work

What is a 3-sigma PEX corner?

- PEX: **P**arasitic **EX**traction
  - Vendor tools: Synopsys StarRC, Calibre xRC, Cadence Quantus QRC, etc.
  - Layout → PEX → SPICE netlist
    - A netlist contains FET instances, passive instances, many parasitic/interconnect resistive elements, and many parasitic/interconnect capacitive elements

- 3-sigma Corner: A corner at which a figure-of-merit reaches its 3-sigma min. or max value.
  - Implicitly, a single SPICE simulation will “magically” reproduce the 3-sigma min. or max. value for the finger-of-merit using a corner model or a netlist at a 3-sigma corner

- 3-sigma R_{max} (or R_{min}) PEX corner: A PEX deck that off-set each metal level’s thickness and width bias by a pre-calibrated and fixed amount. After extraction, the resulting SPICE netlist will reproduce the 3-sigma R_{max} (or R_{min}) value of each metal level in a single SPICE simulation.

- The pre-calibration is typically done for min. design width of each metal level.

- Other 3-sigma PEX corners: 3-sigma R_{min}, 3-sigma C_{max}, 3-sigma C_{min}

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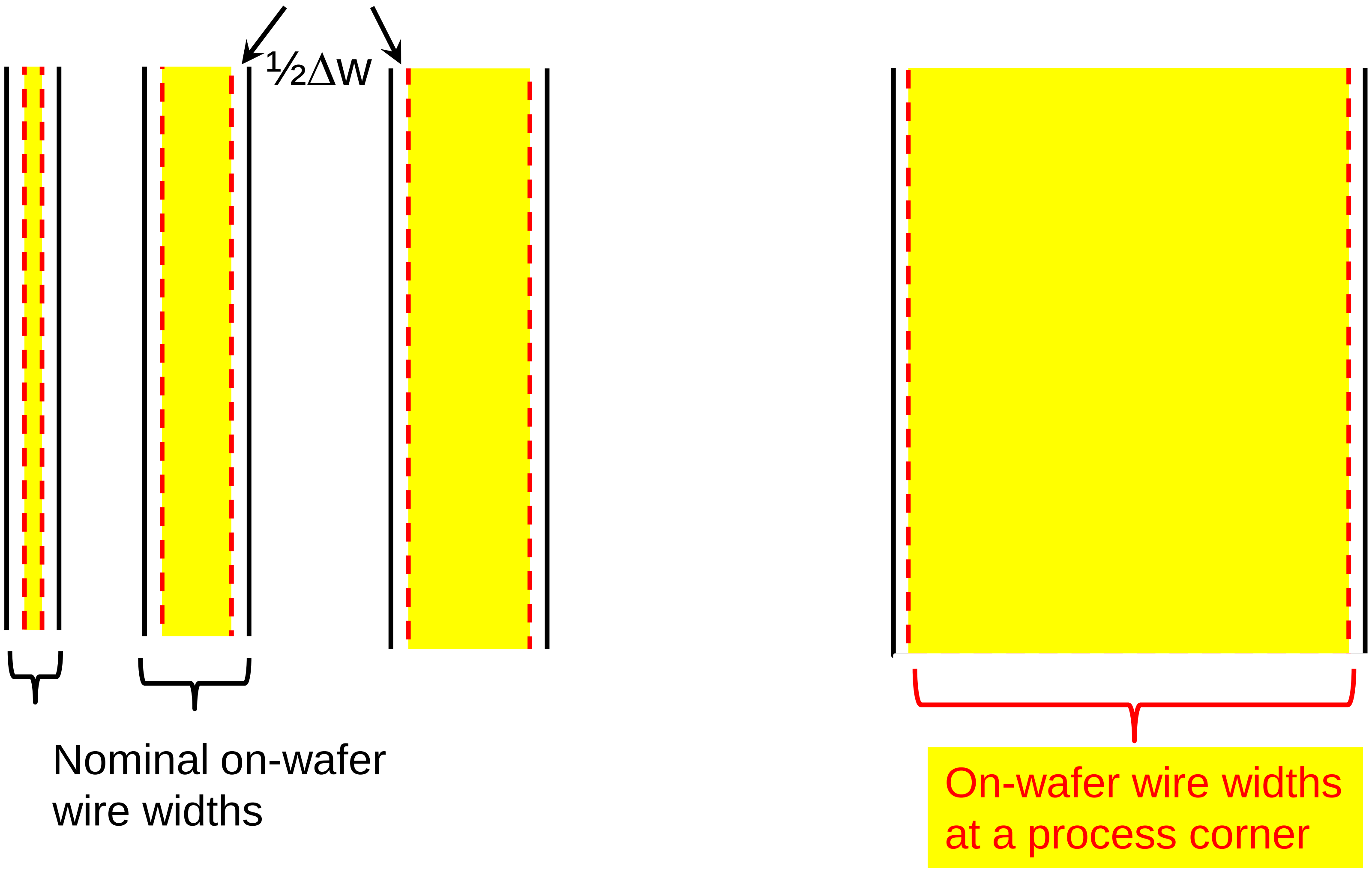
Problem Statement: How to reproduce 3σ wire R_{max} corner values for **all** wire widths with a **single** set of tolerance off-set in width and thickness?



There are random variations in both wire width and thickness.

Δw is the same for all wires of different design widths (global/correlated variations), and so is Δt.

Wire Resistance's Statistical Model

- Examine Wire R's 3-sigma max corner using a linearized wire R model.

$$R(w_0; u_w, u_t) = \frac{\rho L}{(w_0 + \sigma_w u_w)(t_0 + \sigma_t u_t)} \approx R_0 \left(1 - \frac{\sigma_w}{w_0} u_w - \frac{\sigma_t}{t_0} u_t\right)$$

$$R_0(w_0) = \frac{\rho L}{w_0 t_0}$$

u_w and u_t are independent random and normalized variables

$$\langle u_w \rangle = \langle u_t \rangle = 0, \qquad \langle u_w^2 \rangle = \langle u_t^2 \rangle = 1, \qquad \langle u_w u_t \rangle = 0$$

$$\sigma_R(w_0) = \langle (R - R_0)^2 \rangle^{\frac{1}{2}} = R_0 \ (w_0) S(w_0)$$

$$S(w_0) = \sqrt{\left(\frac{\sigma_w}{w_0}\right)^2 + \left(\frac{\sigma_t}{t_0}\right)^2}$$

$$R_{3\text{sig,min}}(w_0) = R_0(w_0) - 3\sigma_R(w_0)$$

$$R_{3\text{sig,max}}(w_0) = R_0(w_0) + 3\sigma_R(w_0)$$

Wire Resistance's 3-Sigma Corners for One Width

- Corner Model: Can reproduce a "figure-of-merit" corner in a single simulation.
- Component form: $R(w_0;\, u_w, u_t) = R_0(w_0) - \sigma_R \left(\frac{\sigma_w}{w_0 S} u_w + \frac{\sigma_t}{t_0 S} u_t \right)$
- Vector form: $R(w_0;\, \mathbf{u}) = R_0(w_0) + \sigma_R(w_0)\, \mathbf{n}(w_0) \cdot \mathbf{u}$ $\qquad \mathbf{u} = (u_w, u_t)$

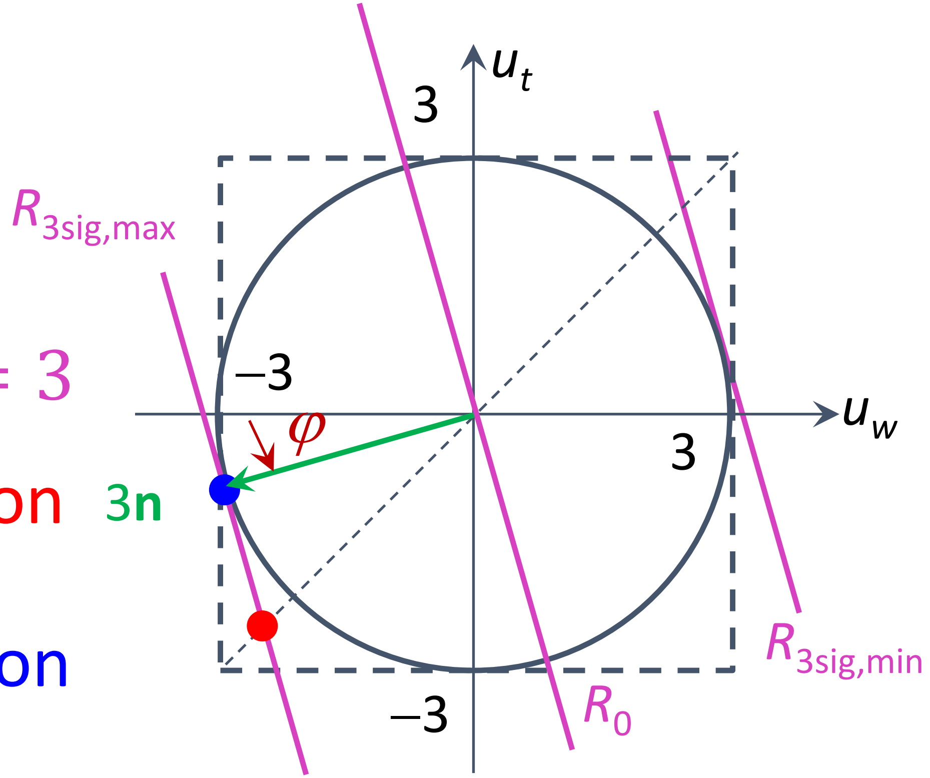
$$\mathbf{n}(w_0) = (-\cos\varphi, -\sin\varphi), \qquad \varphi(w_0) = \arctan\frac{\sigma_t w_0}{\sigma_w t_0}$$

$$0 < \varphi(w_0) < \frac{\pi}{2} \qquad |\mathbf{n}(w_0)| = 1$$

- Many corner solutions for $R_{3sig,max}$: $\mathbf{n}(w_0) \cdot \mathbf{u} = 3$

ex1. $u_w = u_t = \frac{3S}{\frac{\sigma_w}{w_0} + \frac{\sigma_t}{t_0}}$ low-probability solution

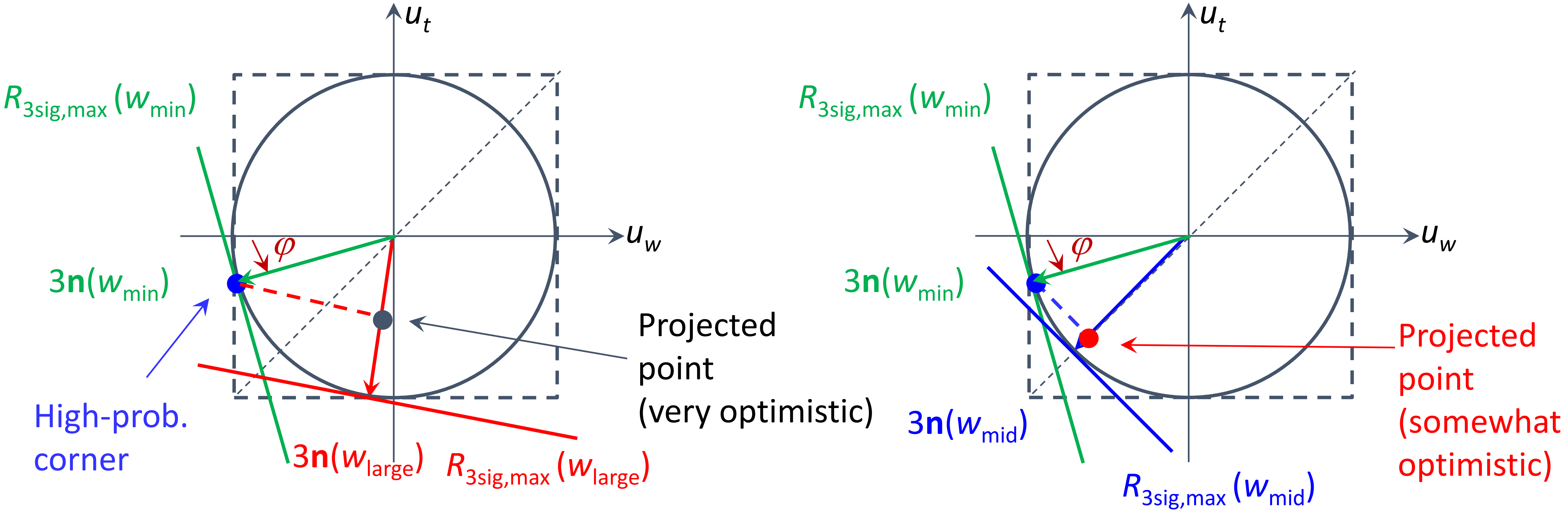
ex2. $\mathbf{u} = 3\mathbf{n}(w_0)$ high-probability solution



W_{min}'s High-Probability Corner for a Larger Width

- Angle φ increases with increasing w_0.
- A 3-sigma corner for one width is not a 3-sigma corner for another width

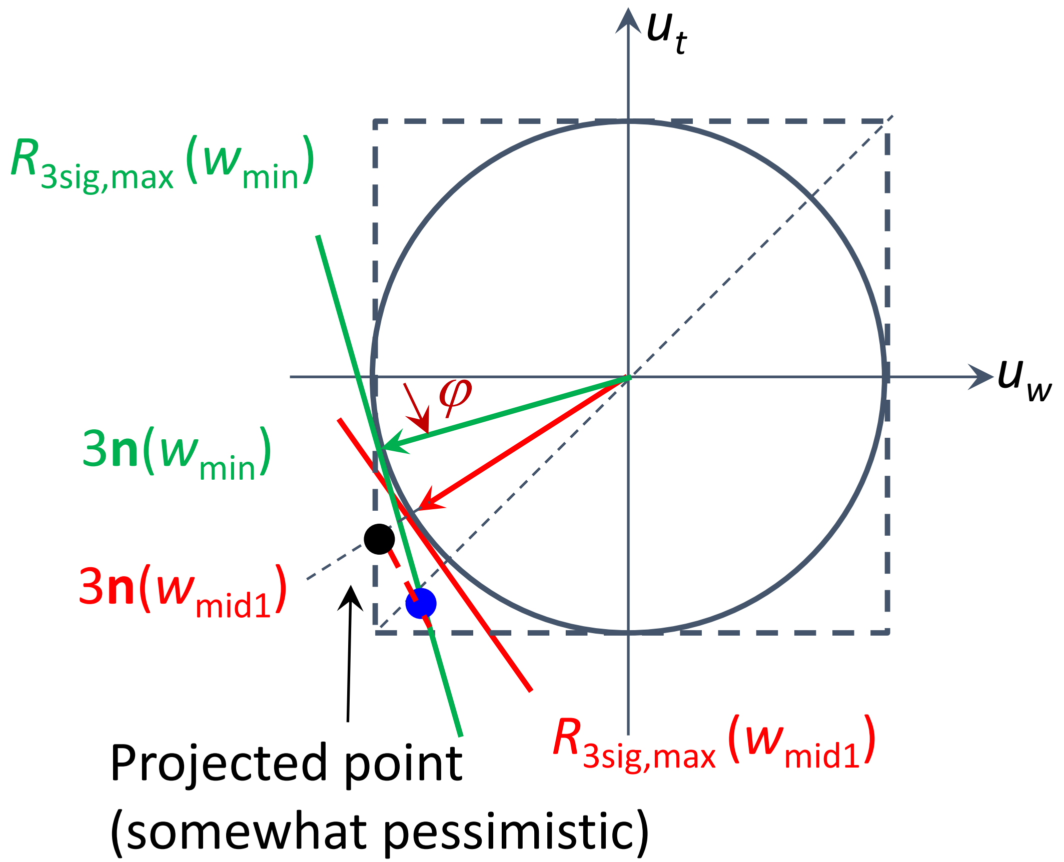
$$R(w_0;\,\mathbf{u}) = R_0(w_0) + \sigma_R(w_0)\,\mathbf{n}(w_0)\cdot\mathbf{u}$$

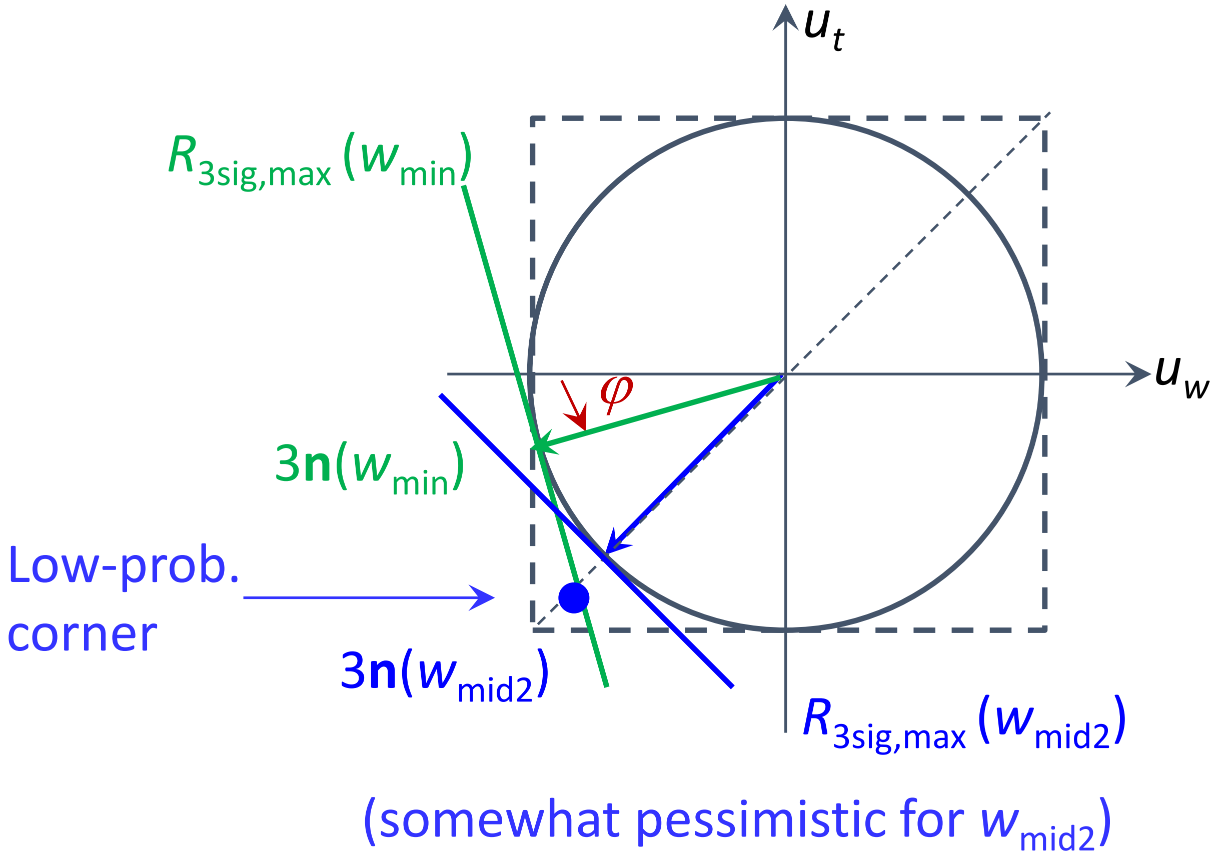


W_{min}'s Low-Probability Corner for a Larger Width

- Angle φ increases with increasing w_0.
- A 3-sigma corner for one width is not a 3-sigma corner for another width

$$R(w_0;\, \mathbf{u}) = R_0(w_0) + \sigma_R(w_0)\, \mathbf{n}(w_0) \cdot \mathbf{u}$$





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Best-Fit 3-Sigma Corners for a Range of Wire Widths [w_{min}, w_{max}]

- A user should provide a width range [w_{min}, w_{max}] for each metal level in his/her circuit.
- Design rules include a min. width and a max. width for each metal level. But, the smaller the range($w_{max} - w_{min}$) is, the more accurate the resulting 3-sigma corner is.

$$\varphi_0 = \arctan\frac{w_{min}\sigma_t}{t_0\sigma_w}, \quad \varphi_1 = \arctan\frac{w_{max}\sigma_t}{t_0\sigma_w}$$

a. Find boundary:

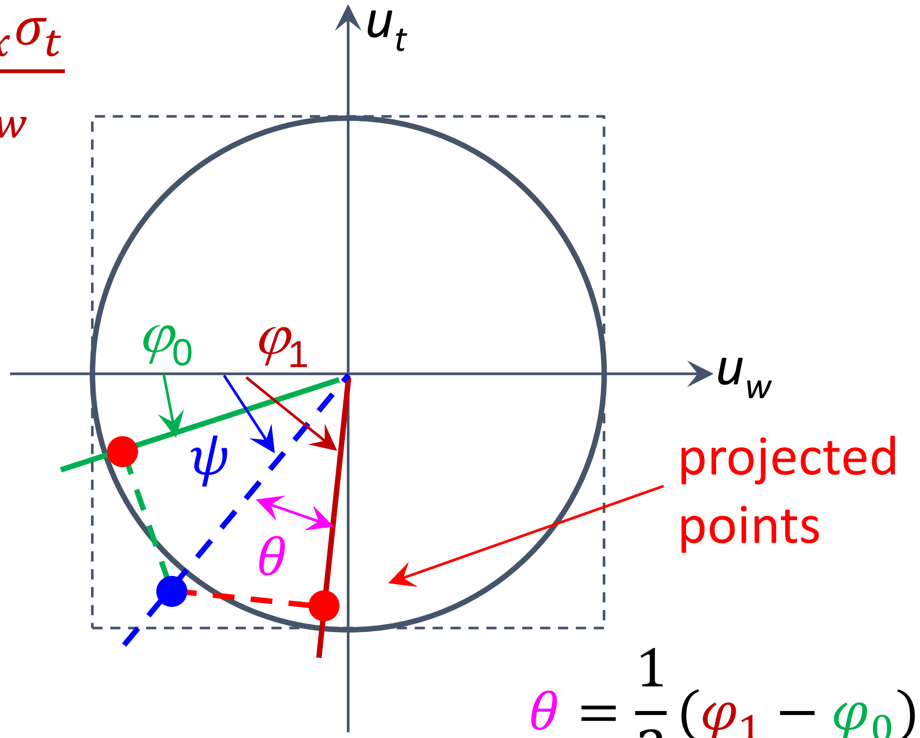
$$\psi = \frac{1}{2}(\varphi_0 + \varphi_1)$$

b. Find center:

$$u_w = \pm\frac{3\cos\psi}{\cos^2(\theta/2)}$$

$$u_t = \pm\frac{3\sin\psi}{\cos^2(\theta/2)}$$

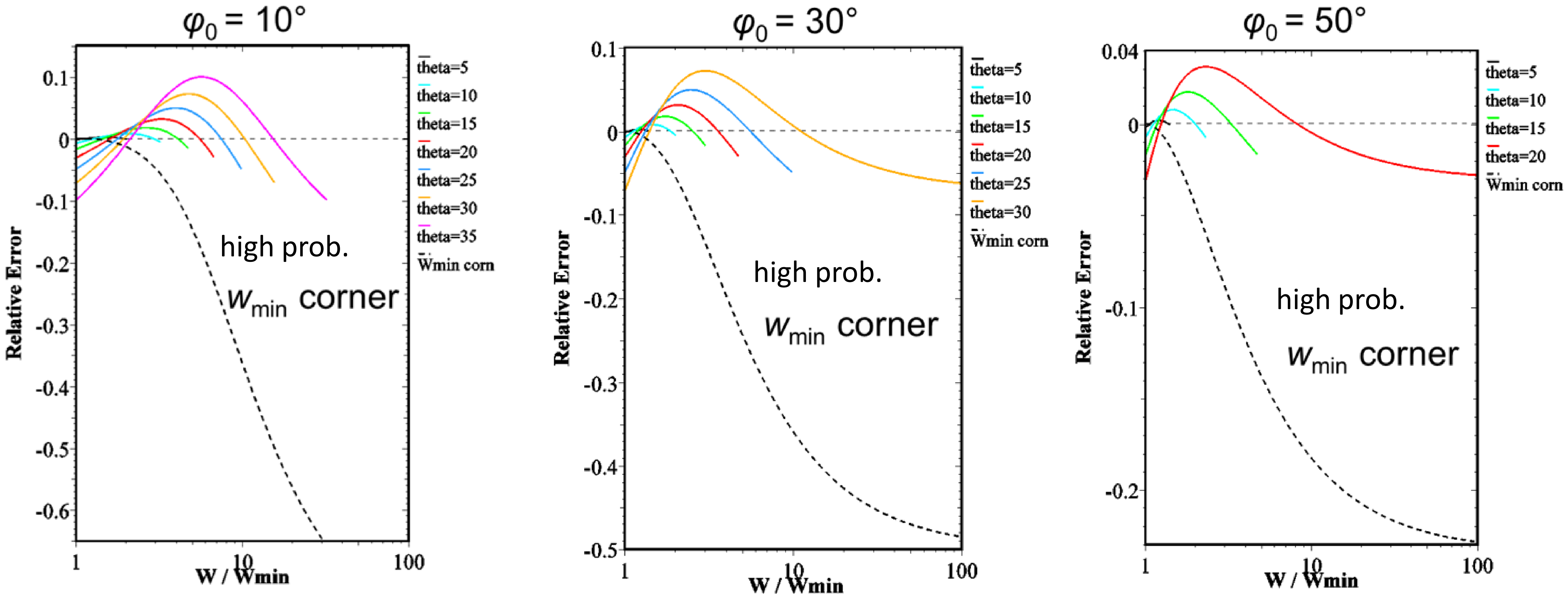
c. Move the blue point outward to minimize the largest distance to the circle for all projected points within the range [φ_0, φ_1].



$$\theta = \frac{1}{2}(\varphi_1 - \varphi_0)$$

+: max. corner; −: min. corner

Improved Corner Model Accuracy for [W_{min}, W_{max}]



Relative error = ($R_{3sig,\text{fixed corner}} - R_{3sig,\text{min/max}}$) / ($3\sigma_R$)

The smaller the angle range 2θ is, the more accurate a single fixed corner will be for all wire width within the width range [W_{min}, W_{max}].

Summary

- We have discussed the inaccuracy issue in semiconductor foundry's 3-sigma PEX corners.
- If a PEX corner is set to reproduce the 3-sigma corner of an interconnect R spec at one wire width, then it does not reproduce 3-sigma R corner at other wire widths in general.
- We propose to generate a best-fit 3-sigma corner for a range of wire width. The smaller the range, the more accurate the 3-sigma corner will be.
- Using a simple diagram, we have shown how to generate best-fit 3-sigma interconnect corners for a given range of wire width.

Future Work

- Interconnect Capacitance: 2 more process parameters (interlevel distances to the metal levels above and below) also affect capacitance. One needs to work in a higher dimensional space to find a concrete solution.

  - Each independent process variation is a one-dimensional space in the corner analysis.